XXI. *On the Phenomena and Laws of Elliptic Polarization, as exhibited in the Action of Metals upon Light.* By David Brewster, *LL.D. F.R.S. Lond. & Edin.*

Read April 22, 1830.

FROM the first dawn of the science of polarization, the action of metals upon light has presented a troublesome anomaly. Malus at first announced that they produced no effect whatever; but by employing a different method of observation, I found that the light reflected by metallic surfaces was so far modified as to produce, when transmitted through thin crystallized plates, the complementary colours of polarized light. From a second series of experiments made previous to mine, Malus came to the conclusion, that the difference between transparent and metallic bodies consisted in this: that the former refract all the light which they polarize in one plane, and reflect all the light which they polarize in another; while metallic bodies reflect what they polarize in both planes.

Having discovered the property of transparent bodies to polarize light by successive reflexions at angles at which a single reflexion produced no perceptible effect*, I resolved to apply this method of examination to metals; and on the 7th of February 1815, when I first made the experiment, I discovered the curious property possessed by silver and gold of dividing a polarized ray into complementary colours by successive reflexions. As this subject promised to open a wide field of inquiry, I prepared for the ardent prosecution of it with all the metallic bodies which could be procured; but the pressure of professional business prevented me for about a month from doing any thing very effectual.

On the 6th of March 1815, I received a letter from M. Biot, requesting some information on a matter of business; and in answering this letter on the same day, I communicated to him an account of the discovery above men-

* Phil. Trans. 1815, p. 142.

tioned*. Immediately after this I received the most perfect plates of silver, one pair polished by friction, and another by hammering; two pair of plates of gold, one of jewellers', and another of fine gold; with plates of steel, platinum, palladium, copper, brass, and speculum metal; and with their help I obtained the general result, that a single reflexion from a metallic surface produces the same effect upon polarized light as a certain thickness of a crystallized body, with many other results, which it is unnecessary here to indicate.

As soon as M. BIOT had received notice of my discovery, he seems to have devoted himself to the same inquiry; and with all the leisure of an Academician, and the splendid apparatus presented to him by the Institute, he obtained many of the results at which I had arrived, and others to which I have no claim; and on the 29th of March he transmitted to me, through Dr. WOLLASTON, an open letter containing an abstract of his experiments, and expressing the hope that they would be of use to me in my researches.

Although this expression led me to believe that I should enjoy the privilege of publishing the first account of my own discovery, yet I took the precaution of having all my papers on the subject signed by the Treasurer of the Royal Society of Edinburgh, and I proceeded with new zeal in the further examination of the subject. I soon learned, however, from M. BIOT, that he meant to treat the subject in his Traité de Physique; and though I remonstrated against this as a breach of courtesy, I had the mortification to see the discovery, to which I perhaps attached too much importance, published for the first time in a foreign work.

I trust the Society will excuse these details as a necessary apology for having so long delayed to fulfil the promise, more than once made in their Transactions, to communicate to them an account of these experiments†. The

* It is related in the History of Optics, Edinburgh Encyclopædia, vol. xv. p. 493, note, that I communicated this discovery to M. BIOT on the day on which it was made:—this is a mistake, as it was done a month afterwards.

† In a letter to Sir JOSEPH BANKS, dated July 28th, 1815, I communicated an abstract of these and other experiments, with a request that he would permit the MS. to remain in his possession, as an evidence of my claims. Sir JOSEPH complied with this request: but nearly two years afterwards, happening to see the MS., he thought that it had been intended for publication, and laid it before the Royal Society without my knowledge. It was accordingly read on the 23rd of January 1817, under the title of Abstract of Experiments on Light, and ordered to be printed. When the proof-sheet was sent me for correction, I requested the paper to be cancelled, as it was not intended for publication.

reasons which I have assigned were subsequently strengthened by new inquiries which at first threw great doubts over the views which M. Biot and I had taken of the subject, and finally convinced me of the rashness of our generalizations. The study of M. Fresnel's fine discoveries respecting circular polarization enabled me to advance still further in the inquiry; and having more recently resumed the investigation, I trust I shall now be able to present to the Society a satisfactory analysis of the singular phenomena exhibited in the action of metals upon light.

Sect. I. *On the action of metals upon common light.*

When we analyse with a rhomb of calcareous spar a ray of common light, reflected at different angles from a metallic surface, there will be observed in one of the images a defalcation of light, as if a portion of the incident ray was polarized in the plane of reflexion. This effect will be still more distinctly seen if we examine the system of polarized rings formed round the axes of crystals by means of the light reflected from metals. If the light had suffered no modification by reflexion, or if the metal reflected in equal quantities the light polarized in opposite planes, the rings would not be visible at all; but it will be found that they are easily seen in the light reflected by all metals. They are most distinctly visible at an incidence of about 74°, at an average, and become fainter and fainter as the incidence exceeds or falls below that angle. They appear best defined in light reflected from galæna and metallic lead, and with least distinctness in light reflected from silver and gold, as shown in the following Table, in which the metals are arranged in the order in which they exhibit the rings most brightly, and consequently in the order in which they polarize the greatest quantity of light in the plane of reflexion.

Galæna,	Antimony,	Bismuth,	Grain tin,
Lead,	Steel,	Mercury,	Jewellers' gold,
Gray cobalt,	Zinc,	Copper,	Fine gold,
Arsenical cobalt,	Speculum metal,	Tin plate,	Common silver,
Iron pyrites,	Platinum,	Brass,	Pure silver.

If we now take two plates of each of these metals and examine the light which has undergone more than one reflexion, we shall find that the quantity of light which each polarizes in the plane of reflexion increases with every reflexion, and that in several of them the whole incident pencil is completely polarized.

When the luminous object is a wax-candle placed at the distance of ten feet, eight reflexions from a plate of steel at angles between 60° and 80° polarize the whole of the light, while at angles above 80° and below 60° a greater number of reflexions is required. With galæna, lead, cobalt, and antimony, a much smaller number of reflexions polarizes the whole pencil; whereas with pure and highly polished silver a very great number is necessary: the light reflected from the silver becomes redder and redder, indicating an increasing absorption or dispersion of the less refrangible rays.

By the use of common light it would be in vain to attempt to discover the law according to which the polarization of the incident pencil is effected in different metals; but by another mode of analysis we shall be led to the mathematical law for computing the exact proportion of the reflected pencil which is polarized at certain angles when the number of reflexions exceeds one.

SECT. II. *On the action of metals upon polarized light.*

If a pencil of polarized light is received on a polished metallic surface placed so as to have a rotatory motion round the polarized ray, the reflected light will receive no modification (excepting what arises from its property of apparently polarizing a portion of light in the plane of reflexion) when the plane of incidence is inclined 0°, 90°, 180°, and 270° to the plane of primitive polarization; but in every other azimuth of the plane of incidence the reflected pencil will be found to have suffered a remarkable change, which gradually increases as the azimuth of that plane varies from 0° to 45°, from 90° to 135°, from 180° to 225°, and from 270° to 315°. At the azimuths of 45°, 135°, 225°, and 315°, the effect is a maximum, and it gradually diminishes from 45° to 90°, from 135° to 180°, from 225° to 270°, and from 315° to 360°.

In order to investigate the nature of this change, we shall suppose the plane of reflexion from the metal to be inclined −45°, or to the left of the plane of primitive polarization. In this position let a plate of highly polished steel receive the polarized ray of ordinary intensity. At 89°, 88°, and 87° of incidence, almost no change is produced upon it by the action of the metal. We can easily see that the plane of polarization of the ray is turned from right to left, exactly as it would be by a transparent surface. In like manner at all angles of incidence from 0° to about 40° no decided effect is produced, except the change in the plane of polarization. At angles less than 87° the change begins to appear, reaches its maximum at about 75°, and diminishes gradually to 40°. By means of the analysing rhomb, it is easily seen that a great portion of the original pencil has had its plane of polarization changed from +45° to 0°, as the incidence diminishes from 75° to 0°. If, indeed, we measure the rotation of the principal section of the rhomb when the extraordinary pencil is a minimum at different angles of incidence, we shall find it to correspond with $45° - \phi$, ϕ being calculated from the formula $\tan \phi = \frac{\cos(i + i')}{\cos(i - i')}$, in which $\frac{\sin i}{\sin i'} = 3.732$, the index of refraction for steel. The value of ϕ will be found to be nearly the same at 87° and 40°, which shows why at these two angles the change under our consideration is just beginning to appear with light of ordinary intensity.

The physical effect of the metallic surface being a maximum at 75°, we shall now examine the character of the pencil reflected at that angle.

1. The pencil thus reflected is not polarized light, because it does not vanish during the revolution of the analysing rhomb.

2. It is not common light, because when we reflect it a second time at 75° from another steel surface, it is restored to light polarized in one plane.

In order to discover its nature, let it be transmitted along the axis of calcareous spar. The system of rings is changed into the form shown in Plate X. fig. 1. almost exactly in the same manner as if a thin film of a crystallized body which polarizes the pale blue of the first order had crossed the system. If we substitute for the calcareous spar films of sulphate of lime, which give different tints, we shall find that these tints are increased according as the metallic action coincides with, or opposes that of the crystal.

On the authority of this experiment I was led to believe that metals acted upon light like crystallized plates; and when I found that the colours were not only better developed, but more pure after successive reflexions, it was a natural though a rash generalization, to conclude as I did, and as M. Biot did after me, that each successive reflexion corresponded to an additional thickness of the crystallized film.

In order to show the incorrectness of this deduction, let a ray polarized $+45°$ be reflected twice from steel at angles of 75°. In this case the effect of the second reflexion should be to double the tint produced by the first, if the tints are those of crystallized plates. The result, however, is, that the whole of the light is polarized in one plane, in place of consisting of two pencils polarized in opposite planes. M. Biot got over this embarrassment by regarding the tint produced by two reflexions as the white of the first order, which, in consequence of its complementary tint being black, is the only one where the light is all polarized in one plane: but had he examined the light reflected four times, six times, or eight times at 75°, he would have still found it all polarized in one plane, a result entirely incompatible with the supposition of the tints rising with the number of reflexions. That the tint is not the white of the first order may be more easily proved by making it pass along the axis of calcareous spar; for we shall find that in place of producing an increment of tint, the effect of the second reflexion has been to destroy entirely the effect of the first, and to restore the ray to common polarized light. All this will appear by the perfection of the system of rings seen through the spar. If we examine in a similar manner the light which has undergone any number of reflexions between the plates, we shall easily ascertain that the effect never exceeds that of a quarter of a tint in Newton's scale.

Having thus ascertained that light polarized $+45°$, and reflected at the maximum polarizing angle of metals, is neither common light nor polarized light, nor light constituted like that which passes through thin crystallized plates, I conceived the idea of its resembling circularly polarized light—that remarkable species of light which comports itself as if it revolved with a circular motion during its transmission through particular media.

According to Fresnel's beautiful discovery, a ray of light polarized $+45°$ is circularly polarized when it has suffered two total reflexions from glass at an angle of $54\frac{1}{2}°$; and when such a ray is made to suffer other two reflexions at the same angle, it is restored to the state of light polarized $-45°$ to the plane of reflexion, whatever be the azimuth of the second plane of reflexion in relation to the first. In like manner I shall proceed to show that a ray of light polarized $+45°$, and reflected once at the maximum polarizing angle from metals and certain metallic ores, has an analogous polarization, viz. a polarization hitherto unrecognized, and intermediate between circular and rectilineal polarization.

Let the ray polarized $+45°$ be reflected at 75° from steel, and let a second plate of steel be made to turn round the ray thus reflected. At the azimuths of 45°, 135°, 225°, and 315°, with the plane of primitive polarization, that is, when the planes of the two reflexions are either coincident or rectangular, the first reflected ray will be restored to polarized light at an incidence of 75°. At azimuths of 0° and 180° the restoration will be effected at an incidence of 80°, while at azimuths of 90° and 270° it will take place at an incidence of 70°, and at intermediate azimuths it will take place at intermediate incidences. Hence the ray of light reflected from steel, though it has the general properties of a circularly polarized ray, differs from it in this remarkable particular, that it requires different angles of incidence in different azimuths to restore the polarized light.

In circular polarization, as we have seen, the ray has the same properties in all its sides; and the angles of reflexion at which it is restored to polarized light in different azimuths are all equal, like the radii of a circle described round the ray. Hence, without any theoretical reference, the term circular polarization is from this and other facts experimentally appropriate. In like manner, without referring to the theoretical existence of elliptic vibrations produced by the interference of two rectilineal vibrations of unequal amplitudes, we may give to the new phenomena the name of elliptic polarization, because the angles of reflexion at which this kind of light is restored to polarized light may be represented by the variable radius of an ellipse.

In circular polarization the restored ray has its plane of polarization always inclined $-45°$ to the plane of the second system of reflexions. In elliptic polarization the difference is remarkable. The inclination of the plane of the restored pencil is likewise −, but always less than 45°, as will appear from the following Table, which contains the greater number of metallic bodies.

Names of Metals.	Angles of Restoration.		Names of Metals.	Angles of Restoration.	
	°	′		°	′
Total reflexions .	45	0	Bismuth	21	0
Pure silver . .	39	48	Speculum metal . .	21	0
Common silver .	36	0	Zinc	19	10
Fine gold . . .	35	0	Steel	17	0
Jewellers' gold .	33	0	Iron pyrites . . .	14	0
Grain tin . . .	33	0	Antimony	16	15
Brass	32	0	Arsenical cobalt . .	13	0
Tin plate . . .	31	0	Cobalt	12	30
Copper	29	0	Lead	11	0
Mercury . . .	26	0	Galæna	2	0
Platina	22	0	Specular iron . .	0	0

The bodies in this Table are obviously in the inverse order according to which they polarize most light in the plane of reflexion.

I have inserted at the top of the Table the inclination of the restored pencil in total reflexions, which is 45°; and at the bottom, that of specular iron, which is 0°; in order to show the transition from elliptic polarization to circular polarization on the one hand, and to rectilineal polarization on the other.

In these experiments the primitive ray was polarized +45° to the plane of reflexion; but when this angle diminishes, the plane of the restored ray approaches to the plane of reflexion, and ultimately coincides with it at 0°; and when this angle increases, the plane of the restored ray recedes from the plane of reflexion, and the two planes form an angle of 180° when the other angle becomes 90°.

The following experiments were made with plates of pure silver, in which the inclination φ was 39° 48′, when the inclination x of the plane of polarization was 45°.

Inclination x of the Plane of primitive Polarization to Plane of Reflexion.	Observed Inclination of the restored Ray to the Plane of Reflexion or φ.	Inclination φ calculated by the Formula.
°	° ′	° ′
+ 90	− 90 0	− 90 0
85	84 36	84 0
75	74 10	72 10
65	63 51	60 46
55	52 18	49 57
45 . . . $\theta =$	39 48	39 48
35	32 23	30 28
25	23 10	21 14
15	13 16	12 35
5	4 40	4 10
0	0 0	0 0

Calling θ the inclination or value of φ at 45°, we may represent these observations by the formula, $\tan \varphi = \tan \theta \tan x$, and the actual change of the plane of polarization, or R, will be $R = x + \varphi$.

When φ is given, $\tan x = \frac{\tan \phi}{\tan \theta}$, and when $\varphi = 45°$, and consequently $\tan \varphi = 1$, we have, $\cot x = \tan \theta$, and $x = 90° - \theta$.

Since light polarized +45° is elliptically polarized by one reflexion from steel at 75°, and is restored to light polarized −17° by a second reflexion at 75°, it is clear that a third reflexion at 75° will again polarize it elliptically, while a fourth reflexion at 75° will again restore it to light polarized $+\varphi$, φ being a quantity less than 17°, and given by the preceding formula. The same effects will be reproduced with different numbers of reflexions, as in the following Table.

No. of Reflexions from Steel at 75° of Incidence.	State of the Light Reflected.	Inclination of the Plane of Polarization. Observed.	Calculated.
		° ′	°
1	Elliptically polarized.		
2	Restored to light polarized	− 17 0	− 17

No. of Reflexions from Steel at 75° of Incidence.	State of the Light Reflected.	Inclination of the Plane of Polarization. Observed.	Calculated.
3	Elliptically polarized.		
4	Restored to light polarized	+5° 10′	+5° 22′
5	Elliptically polarized.		
6	Restored to light polarized	−2 0	−1 38
7	Elliptically polarized.		
8	Restored to light polarized	0 0	+0 30
9	Elliptically polarized.		
10	Restored to light polarized	0 0	−0 9
11	Elliptically polarized.		
12	Restored to light polarized	0 0	+0 3

Hence it follows, that at every odd number of reflexions at the maximum polarizing angle the light is elliptically polarized, and at every even number it is restored to a single plane of polarization. In circular polarization the inclination φ of this plane is always $\mp 45°$, even after fifty reflexions, as I have ascertained by direct experiment; but in elliptical polarization the inclination diminishes at every restoration; and in the case of steel it is reduced to near 0° after eight reflexions, when the light is all polarized in the plane of reflexion; that is, the elliptic polarization gradually diminishes and terminates in rectilineal polarization.

The value of φ, as given in the preceding Table, and consequently the number of reflexions when it approaches to 0°, may be deduced from the formula,

$$\tan \varphi = \tan \theta \,.\, \tan x.$$

After the first reflexion $x = +45°$, and φ, or the inclination of the plane of the ray as restored by the second reflexion, is $= -17°$, as given by experiment. Hence the light which suffers the third reflexion, and is thereby elliptically polarized, is not, as originally, polarized $+45°$, but only $-17°$; and consequently, when it is restored after the fourth reflexion, the value of φ must be such as corresponds to an equality in the values of x and θ, both of them being $= 17°$. Hence the formula becomes,

$$\tan \varphi = \tan^2 x, \text{ or } \tan \varphi = \tan^n x;$$

n being the number of pairs of reflexions, or half the number of reflexions which the restored ray has undergone. In this way the last column of the preceding Table has been calculated. The same formula represents also, as it should do, the phenomena at the limits of elliptic polarization. In the case of circular polarization, where the plane of polarization of the restored ray is 45°, we have,

$$x = 45°, \tan x = 1, \text{ and } \tan \varphi = \tan^n x = 1, \text{ or } \varphi = 45°$$

after any number of reflexions however great. In like manner, in rectilineal polarization, where $x = 0°$, we have $\varphi = 0°$, that is, the ray is polarized in the plane of reflexion.

The above formula is suited to any series of reflexions at any angle when the value of φ for the first term of the series is known. The value of φ for two reflexions, the first term of the principal series, can be determined only by experiment, and has been given in a former Table for several metals; but we may determine from it the value of φ for the first term of any other series, provided it is an even number, in the following manner. Making $x =$ the inclination for two reflexions at the maximum polarizing angle, and φ the value of x at any number of reflexions $2\,n$, we shall have,

$$\tan \varphi = \frac{\tan x + \tan^n x}{2}; \qquad \text{(A)}$$

where $\tan {}^n x$ is the value of φ at the maximum polarizing angle for $2\,n$ reflexions; but as no odd number can occur in the principal series, the preceding rule will not apply to such numbers.

The following Table shows the coincidence between the formula and experiment.

SILVER.

Number of Reflexions.	Values of *n*.	Angle of Incidence.	Inclination of Plane of Polarization. Observed.	Calculated.
		° ′	° ′	° ′
2	1	73 0	39 48	39 48
4	2	82 30	37 45	37 22
6	3	85 6	35 0	35 22

STEEL.

Number of Reflexions.	Values of *n*.	Angle of Incidence.	Observed.	Calculated.
2	1	75 0	17 0	17 0
4	2	83 30	11 30	11 17
6	3	85 45	9 30	9 30

When the number of reflexions which begin the series is odd or fractional, we must determine, by the preceding formula, the value of ϕ for the even number immediately above it: and calling ν the number of odd or fractional reflexions, and N the number of even reflexions immediately above ν, ϕ the inclination for N reflexions as given by the formula (A), and ϕ the inclination required, we shall have,

$$\tan \phi' = \tan x - (\nu - 2 \left(\frac{\tan x - \tan \phi}{N - 2}\right). \qquad \text{(B)}$$

The truth of this formula will appear from the following Table.

SILVER.

Number of Reflexions.	Angles of Incidence.	Inclination of the Plane of Polarization. Observed.	Calculated.
	° ′	° ′	° ′
3	79 40	38 28	38 33
5	77 13	33 10	33 36
5	84 5	26 0	26 24

STEEL.

3	77 37	13 15	14 11
5	84 38	10 30	10 23

The same results will be obtained at the angles of equal phase below the maximum polarizing angle.

This last rule is suited to even as well to odd numbers of reflexions, but it does not give precisely the same results for even numbers as the formula (A). The difference, however, is far within the limits of the errors of observation. The inclination, for example, at 4 reflexions, is by formula (A) 37° 22′ for silver, whereas by formula (B) it is 37° 34′, the difference being only 12 minutes.

In circular polarization, therefore, the plane of polarization of the restored light continues by successive reflexions to oscillate on each side of the plane of reflexion with a never varying amplitude from +45° to −45°; while in elliptical polarization, the same plane oscillates with an amplitude continually diminishing till it is brought to nothing in the plane of reflexion.

In steel, as we have seen, the polarization is highly elliptical, and the amplitude of the oscillations of the plane of restoration is quickly brought to zero;

but in silver, where the polarization approaches nearly to circular, the oscillations diminish very slowly in amplitude, as the following Table shows.

No. of Reflexions from Silver at 73° of Incidence.	State of the Reflected Light.	Inclination of the Plane of Polarization, or φ. Observed.	Calculated.
1	Elliptically polarized.		
2	Restored to light polarized	−38° 15′	−38° 15′
3	Elliptically polarized.		
4	Restored to light polarized	+31 15	+31 52
5	Elliptically polarized.		
6	Restored to light polarized	−26 0	−26 6
12	Restored to light polarized		+13 30
18	Restored to light polarized		− 6 42
36	Restored to light polarized		+ 0 47

Owing to the high dispersive power of silver, I found it difficult to carry the comparison any further with white light, as the colours closed in upon the points of evanescence, and rendered it impossible to determine with any precision the inclination of the plane of polarization.

The preceding results afford the clearest explanation of the phenomena which steel and silver exhibit in the reflexion of common light. As common light is similar to two equal pencils polarized +45° and −45°, and as steel brings two such pencils into a state of parallelism with the plane of reflexion, common light must therefore be wholly polarized in the plane of reflexion after 8 reflexions. In like manner we see why the same effect is not produced by silver, because after 8 reflexions the two planes of the pencils are inclined 42°, so as to form a partially polarized pencil.

The same results also furnish us with a method of computing the proportion of polarized light in any pencil of common light, reflected from metals at angles at which the restoration of the elliptical polarized pencil is effected. In order to determine this proportion for steel after two reflexions at 75°, we must consider that a pencil polarized +45° is restored by these two reflexions to light polarized −17°, and consequently a pencil polarized −45° to light polarized +17°. Hence a beam of common light will consist after two reflexions of two pencils +17° and −17° of equal intensity, and consequently in the same state of partial polarization as if common light had been reflected either at an angle of 45° or 68° from a surface of glass. Consequently in the formula *

$$Q = 1 - 2 \sin^2 \varphi, \quad \text{we have } \varphi = 17° \text{ and } Q = 0.829.$$

Hitherto we have considered elliptical polarization as produced only at the maximum polarizing angle. It may be produced, however, by a sufficient number of reflexions at any given angle either above or below the maximum polarizing angle, as appears from the following Table, in which the reflexions are made from two parallel plates of steel.

No. of Reflexions from Steel at which Elliptic Polarization is produced.	No. of Reflexions at which the Pencil is restored to a single Plane.	Angles of Incidence.	
		Calculated.	Observed.
3, 9, 15, &c.	6, 12, 18, &c.	85° 45′	86° 0′
$2\frac{1}{2}$, $7\frac{1}{2}$, $12\frac{1}{2}$, &c.	5, 10, 15, &c.	84 38	84 0
2, 6, 10, &c.	4, 8, 12, &c.	83 30	82 20
$1\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, &c.	3, 6, 9, &c.	79 39	79 0
1, 3, 5, &c.	2, 4, 6, &c.	75 0	75 0
$1\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, &c.	3, 6, 9, &c.	68 53	67 40
2, 6, 10, &c.	4, 8, 12, &c.	60 2	60 20
$2\frac{1}{2}$, $7\frac{1}{2}$, $12\frac{1}{2}$, &c.	5, 10, 15, &c.	56 5	56 25
3, 9, 15, &c.	6, 12, 18, &c.	51 24	52 20

The numbers given in the third column are calculated by the following method. The relation of the preceding phenomena to the angle of maximum polarization is obvious; and if we consider the nature of the formula, $\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}$, we shall see that the angles at which the rectilineal polarization of the primitive pencil is destroyed have a reference to the rotation which the reflecting surface produces in the plane of polarization. The angles indeed in the third column, at which similar effects are produced above and below 75°, are those at which φ has equal values. This is a very important relation, and enables us to determine the phase P of the two inequal portions of oppositely polarized light, by the interference of which the elliptic polarization is produced. It may be expressed by $\quad P = 2\,R.$

But $\quad R = 45° - \varphi,$

Hence $\quad P = 90° - 2\,\varphi,$

$$\tan \varphi = \frac{\cos(i + i')}{\cos(i - i')}.$$

* See my Paper "On the Law of the Partial Polarization of Light by Reflexion," *supra*, p. 76.

In this manner we obtain the following results.

No. of Reflexions for Elliptic Polarization.	Angle of Incidence on Silver.	Angle of Incidence on Steel.	Inclination of Plane, or ϕ.	Rotation of Plane, or R.	Phase, or P.
	° ′	° ′	° ′	° ′	° °
3	85 6	85 45	30 0	15 0	30 = $\frac{1}{3}$ of 90
2½	83 49	84 38	26 15	18 45	37½ = $\frac{5}{12}$ of 90
2	82 30	83 30	22 30	22 30	45 = $\frac{1}{2}$ of 90
1½	78 8	79 39	11 15	33 45	67½ = $\frac{3}{4}$ of 90
1	73 0	75 0	0 0	45 0	90 = $\frac{1}{1}$ of 90
1½	66 25	68 53	11 15	33 45	67½ = $\frac{3}{4}$ of 90
2	57 16	60 2	22 20	22 30	45 = $\frac{1}{2}$ of 90
2½	53 17	56 5	26 15	18 45	37½ = $\frac{5}{12}$ of 90
3	48 38	51 24	30 0	15 0	30 = $\frac{1}{3}$ of 90

In the results of the two preceding Tables, where the number of reflexions is an integer, it is easily understood how an elliptically polarized ray begins to retrace its course, and recover its state of polarization in a single plane, by the same number of reflexions by which it lost it: but it is interesting to observe, when the number of reflexions is 1½, 2½, 3½, that the ray must have acquired its elliptic polarization in the middle of the second and the third reflexion; that is, when it had reached its greatest depth within the metallic surface. It then begins to resume its state of polarization in a single plane, and recovers it at the end of 3, 5, and 7 reflexions. This stationary point at which the retrograde effect commences, may be made to have its position at any depth beneath the surface, by changing the angles of some of the reflexions, or by combining plates of metal of different polarizing powers.

The same curious property is exhibited in total reflexions, as I have found that the circular polarization can be produced by 2½, 3½, &c. reflexions.

Hitherto we have chiefly examined the phenomena when the reflexions are performed either all above or all below the polarizing angle. We shall now proceed to the case when one reflexion is made on one side, and one on the other side of the maximum polarizing angle.

When a ray polarized + 45° has been reflected once from steel at an angle of 85° or of 54°, it has acquired partially the state of elliptic polarization, and to such a degree that three reflexions more at the same angle will complete the effect. But if the ray partially polarized elliptically by one reflexion at 85° suffers a second reflexion at 54°, it does not acquire more elliptic polarization, but it retraces its course, and recovers its state of single polarization. The same phenomenon occurs at the following angles.

Angles of partial Elliptic Polarization.		Values of φ.		Angles at which it recovers its Polarization.		Values of φ.	
	°	°	′		°	°	′
1 Reflex. at	$87\frac{1}{2}$	36	5	1 Reflex. at	41	36	11
———	85	27	28	———	54	28	0
———	80	12	12	———	68	12	36
———	77	5	24	———	72	5	59
———	75	0	0	———	75	0	0

It is obvious, by comparing these angles with those in the preceding Table, that they correspond, and are those at which equal phases or rotations are produced.

The effect of two reflexions, at angles of equal phase, upon the inclination I of the plane of polarization is shown in the following Table.

				Inclination I of the Plane of Polarization. Observed.	Calculated.	
	°		°	°	°	′
1 Reflex. at	90	and 1 at	0	45	45	0
———	$87\frac{1}{2}$	———	41	—	30	0
———	85	———	54	26	26	5
———	80	———	68	19	20	8
———	77	———	72	18	17	2
———	75	———	75	17	17	0

The last column of the table is calculated by the formula

$$I = \tan \varphi\,(45^\circ - i') + i',$$

i being 17°, or the inclination after two reflexions at the maximum polarizing angle.

In the preceding inquiry we have considered only the phenomena when the consecutive reflexions are performed in coincident planes. The investigation becomes more troublesome, and the results more interesting when the plane of the second reflexion is presented in every different azimuth to the ray that is either wholly or partially elliptically polarized by the first reflexion.

Let a pencil be elliptically polarized by one reflexion from steel at 75°, and let the azimuths be reckoned from the plane of this reflexion. We have already seen that a second reflexion at 75° in azim. 0° and 180° restores the pencil to a single plane of polarization; but if we turn the plane of the second reflexion into azim. 45° or 225°, we shall find that the angle of restoration is no longer 75°, but 78°. At azim. 90° and 270° it is again 75°, and in azim. 135° and 315° it is only 68°, having varied from 68° to 78°.

The following Table shows the observed and calculated angles of restoration in different azimuths.

Azimuths from Plane of first Reflexion.		Angles of Restoration from Steel.	Complement of Angles of Restoration, or Elliptical Radii. Observed.	Calculated.
0°	and 180°	75°	15°	14°.9
$22\frac{1}{2}$	$202\frac{1}{2}$	77	13	12.7
45	225	78	12	12
$67\frac{1}{2}$	$247\frac{1}{2}$	$77\frac{3}{4}$	$12\frac{1}{4}$	12.7
90	270	75	15	14.9
$112\frac{1}{2}$	$292\frac{1}{2}$	70	20	19
135	315	68	22	22
$157\frac{1}{2}$	$337\frac{1}{2}$	70	20	19
180	360	75	15	14.9

The radii in the two last columns are obviously those of a curve approaching to an ellipse whose major and minor axes are situated, the one 45° to the right, and the other 45° to the left of the plane of the first reflexion. The major semiaxis is 22°, and the minor 12°. Hence calling x the variable radius of the ellipse, a the greater and b the lesser semiaxis, and θ the azimuth, reckoned from the lesser axis, in which the radius x is wanted, we shall have

$$x = \frac{a\,b}{\sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta}}.$$

When $\theta = 45°, 135°$, &c. $\sin^2 \theta \cos^2 \theta = \frac{1}{2}$ and $x = \frac{a\,b}{\sqrt{\frac{1}{2}a^2 + \frac{1}{2}b^2}}$.

By calculating the values of x corresponding to the azimuths in the Table, we obtain the numbers in the last column, which are so near the observed numbers as to leave no doubt that an ellipse represents the observations.

If we perform the same experiments with a plate of silver at 73°, we shall observe, with surprise, that the angle of restoration is the same in all azimuths, that is, that the ellipse has merged into the circle. There is a slight deviation indeed, just sufficient to show that the circle is slightly oval, but I could not measure the amount of it.

This result arises from the elliptical polarization of silver being very nearly circular. If we call β the angle of restoration after two reflexions, the ratio of a to b, the major and minor axes of the ellipse, may be thus expressed: $a : b = \sin 2\beta : \text{rad}$.

In steel, where $\beta = 17^\circ$ and $2\beta = 34^\circ$, we have $a : b = 0.559 : 1 = 12 : 21\frac{1}{2}$, differing very little from 12 : 22 the actual ratio.

In silver, where $\beta = 39^\circ\ 48'$, $a : b = 0.9835 : 1 = 17 : 17\frac{1}{4}$.

In circular polarization, where $\beta = 45^\circ$, $a : b = 1 : 1$, which gives a circle.

In rectilineal polarization, where $\beta = 0$, $a : b = 0 : 1$, which gives a straight line.

It now becomes an interesting subject of inquiry to ascertain the form and position of the ellipse, when the angle of incidence on the first plate exceeds or falls below the maximum polarizing angle.

The following experiments were made with silver at angles of incidence of 80° and 68°, the maximum polarizing angle being 73°.

Silver.—Angle of Incidence on First Plate 80°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
°	° ′	°	° ′
0	28 2	0	28 2
$11\frac{1}{4}$	26 35	$11\frac{1}{4}$	24 40
$22\frac{1}{2}$	25 20	$22\frac{1}{2}$	21 0
$33\frac{3}{4}$	21 13	$33\frac{3}{4}$	16 40
45	18 20	45	14 35
$56\frac{1}{4}$	14 20	$56\frac{1}{4}$	11 10
$67\frac{1}{2}$	11 32	$67\frac{1}{2}$	10 0
$78\frac{3}{4}$	10 15	$78\frac{3}{4}$	10 0
90	10 0	90	10 0

Silver.—Angle of Incidence on First Plate 68°.

°	°	°	°
0	13	0	13
$11\frac{1}{4}$	14	$11\frac{1}{4}$	13
$22\frac{1}{2}$	$15\frac{1}{3}$	$22\frac{1}{2}$	$13\frac{1}{2}$
$33\frac{3}{4}$	16	$33\frac{3}{4}$	14
45	17	45	$14\frac{1}{2}$
$56\frac{1}{4}$	19	$56\frac{1}{4}$	$15\frac{1}{2}$
$67\frac{1}{2}$	20	$67\frac{1}{2}$	$16\frac{1}{2}$
$78\frac{3}{4}$	20	$78\frac{3}{4}$	18
90	20	90	20

In the first of these sets of experiments, the semiaxes of the ellipse are as 10°

to 28°, and its major axis is in azim. 0° and 180° or in the plane of the first reflexion.

In the second series the ratio of the semiaxes is as 13° to 20°, and the major axis is in azim. 90° and 270°, or perpendicular to the plane of the first reflexion; but in both series there is a want of symmetry in the curve to the right of azim. 0° where it bulges out, showing that in both series the greater axis is a little to the right of azim. 0°.

Hence it appears that in silver, whose elliptic polarization is nearly circular, the ellipse which regulates the angles of restoration has its greater axis in the plane of the first reflexion for all angles greater than 73°, the maximum polarizing angle; and from a circle it increases in ellipticity till at the limit of 90° the lesser semiaxis is 0°, and the greater 90°, and it becomes a straight line. For angles above 73° the ellipse has its greater axis perpendicular to the plane of reflexion, and gradually increases in ellipticity from the circle till at the limit of 0° its lesser semiaxis is 0°, and its greater 90°, when it becomes a straight line.

The peculiar character of elliptic polarization shows itself in another manner, and with peculiar interest, in the variable position of the ellipses which regulate the angles of restoration upon steel.

We have already seen that the curve which is circular in silver at the maximum polarizing angle, is in steel an ellipse whose semiaxes are as 12° to 22°, the greater axes being inclined 45° to the right of azim. 0°.

The following Table will show how the effect varies at angles of incidences above and below the polarizing angle.

STEEL.—Angle of Incidence 80°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
° 0	° 23	° 0	° 23
11¼	25	11¼	20
22½	26	22½	16⅓
33¾	24	33¾	13
45	20½	45	11½
56¼	18	56¼	10
67½	15⅓	67½	9½
78¾	11	78¾	9¾
90	10	90	10

STEEL.—Angle of Incidence 68°.

Azimuth to Right.	Complement of Angle of Restoration by Second Plate.	Azimuth to Left.	Complement of Angle of Restoration by Second Plate.
0°	11°	0°	11°
$11\frac{1}{4}$	24	$11\frac{1}{4}$	10
$22\frac{1}{2}$	$24\frac{1}{2}$	$22\frac{1}{2}$	9
$33\frac{3}{4}$	$25\frac{1}{2}$	$33\frac{3}{4}$	$9\frac{3}{4}$
45	$26\frac{1}{3}$	45	$11\frac{1}{2}$
$56\frac{1}{4}$	$25\frac{1}{3}$	$56\frac{1}{4}$	15
$67\frac{1}{2}$	20	$67\frac{1}{2}$	18
$78\frac{3}{4}$	21	$78\frac{3}{4}$	20
90	22	90	22

By comparing these results with those obtained from steel at 75°, and with the observations already made on the passage of the ellipse into a straight line, the following results may be deduced.

Angle of Incidence on first Steel Plate.	Ratio of Semiaxes of the Ellipse.	Character of the Ellipse.	Position of the greater Axis of the Ellipse.
0°	0° : 90°	Straight line.	Azim. 90° and 270°
68	9 : 26	Ellipse	—— betw. 45° and 56° to R.
75	12 : 22	Ellipse	—— . . . . . . 45° to R.
80	$9\frac{1}{2}$: 26	Ellipse	—— . . . . . . $22\frac{1}{2}$° to R.
90	0 : 90	Straight line.	—— 0

Hence it is obvious that the major axis of the ellipse is $45° \mp \phi$ R to the right of 0° of azimuth, ϕ being computed from the formula

$$\tan \phi = \frac{\cos (i + i')}{\cos (i - i')}.$$

There is a deviation at the incidence of 68° and 80° of some amount, but still it is scarcely without the limits of the errors of observations when common light is used. In strong lights the coincidence will doubtless be more perfect.

The best method of determining the position of the major axis, is to place the second plate at such an angle to the ray received from the first, that it may exceed by two or three degrees the angle of restoration in azim. 0°. Hence if we turn the second plate round the ray into all azimuths from 0° to 90° in the right hand quadrant where the greater axis lies, it must come into two azimuths where the restoration takes place at the same incidence. The complements of these two angles of incidence will be equal radii of the ellipse, and consequently the azimuth which bisects the two azimuths in question, will be that of the major axis of the ellipse. By increasing the angle of incidence on the second plate, other two azimuths containing equal radii of the ellipse will in like manner be found; and we might, if necessary, at last obtain an angle of incidence where the two radii coincided with the greater axis.

The position of the ellipse being thus given, we may determine it for all angles of incidence. Calling x the angle of incidence on the first plate, then we shall have four points in the ellipse as follows. The radii in azim. 90° and 270° are always $90° - x$, and the radius in azim. 0° and 180° is the complement of the angle of incidence at which φ in the last equation has the same value as at the angle x. Hence the form of the ellipse is also given.

In these experiments the polarization of the primitive ray has always been +45°. When this plane varies its position, that of the restored ray also changes, as we have already shown; but it remains to be seen what change takes place in the angles of restoration. At all angles of incidence, a variation in the plane of primitive polarization does not alter the angles of restoration or the corresponding radii of the ellipse in azim. 0°, 90°, 180°, and 270°; but at all intermediate azimuths of the second plate the angles of restoration diminish while the primitive plane varies from 45° to 0°, and increase when it varies from 45° to 90°. The following experiments show the progress of the change when the azimuths of the second reflexion are +45° and −45°.

STEEL.

Inclination of Plane of prim. Polarization.	Azimuth of Second Reflexion +45° to Right.		Azimuth of Second Reflexion −45° to Left.	
	Observed.	Calculated.	Observed.	Calculated.
°	°	° ′	°	° ′
0	0	0 0	1	0 0
5	2	2 4	2	1 0
10	4	3 46	3½	2 3
15	6	5 43	4½	3 7
20	8½	7 50	6	4 14
25	11	9 54	6	5 26
30	13	12 11	7	6 43
35	15	14 40	8	8 6
40	18	17 25	9½	9 41
45	20½	20 30	11½	11 30

These observations are represented by the formula, $\tan \theta = \tan a \cdot \tan x$; a being the angle of restoration when θ, the inclination of the plane of primitive polarization, is 45°.

I have not given the values of θ from 45° to 90°, because it is difficult to ascertain even in strong lights when the evanescence commences. At 90° the action of the first plate is 0, so that at this limit the angle of restoration is the angle at which the elliptic polarization is no longer visible, from the smallness of the angle of incidence, an angle which varies with the intensity of the light employed.

Hitherto we have attended only to the phenomena produced by two similar metals. When the metals are dissimilar, the one silver and the other steel, I found that at the mean maximum polarizing angle of 74°, the inclination of the plane of the restored ray was 28° 30′. But $28° \, 24' = \frac{39° \, 48' + 17°}{2}$, so that the inclination is an arithmetical mean between that of silver and that of steel. By four reflexions at 74° the inclination was reduced to 14°, while by four reflexions at about 83° and 58° the inclination was $21\frac{1}{2}°$, nearly equal to $\frac{28° \, 30' + 14°}{2}$, according to the formula in page 297. By thus combining dissimilar metals we may produce elliptic polarization of all degress of intensity intermediate between those produced by similar metals.

As the circular polarization of total reflexion is the limiting case of elliptical polarization, it becomes important to establish by experiment their intimate connexion and almost perfect similarity. Upon combining metallic and total reflexions this was at once evident; and I found in general that circular polarization of any intensity, as produced by either one or more reflexions from glass, may always be restored to rectilineal polarization by one or more metallic reflexions, provided the latter are all made at angles less than the maximum polarizing angle, and that the two classes of reflexions are performed in coincident planes.

As this takes place throughout the whole range of total reflexion from 41° to 90°, it follows that total differs from metallic reflexion in its not having two opposite kinds of circular polarization, like the two opposite kinds of elliptical polarization which take place on each side of the maximum polarizing angle of metals. But notwithstanding this, the circular like the elliptic polarization has a maximum at about 50°, declining rapidly to zero at 41°, and on the other side slowly to zero at 90° of incidence.

When one reflexion from steel was combined with two total reflexions from glass at $54\frac{1}{2}°$, the inclination of the plane of the restored ray was $30\frac{1}{2}°$, an arithmetical mean between 45° that of total reflexion, and 17° that of steel, for $\frac{45° + 17°}{2} = 31°$. With silver the inclination was $42\frac{1}{2}°$, and $\frac{45° + 39°\ 48'}{2} = 42°\ 24'$.

If we make the metallic reflector receive the circularly polarized ray in every azimuth, we shall find that in azimuth 90° the circular polarization is compensated by a metallic reflexion above 80°. As the azimuth diminishes to 0°, this angle of compensation diminishes also, passes through 75° in the case of steel, and diminishes to a number depending on the angle of incidence at which the total reflexion is made. We are thus enabled to study the phenomena of circular polarization by the aid of metals, and to obtain results at which it would be exceedingly difficult, if not impossible, to arrive by any other method. This subject, however, presents too wide a field to be treated thus incidentally.

SECT. III.—*On the complementary colours produced by successive reflexions from the polished surfaces of metals.*

I have already given a general account of the phenomena of colour produced by successive reflexions; and I have shown that the tints thus produced are by no means the same as those of crystallized plates, as they do not rise in the scale by successive reflexions.

In my early experiments on total and metallic reflexions, I regarded the two classes of phenomena as exactly the same, mutatis mutandis; and in communicating these results to Dr. YOUNG, I pointed out their coincidence with his theoretical views. Dr. YOUNG noticed these experiments in the following manner*.

"Dr. BREWSTER has also shown that the total reflexion of light within a denser medium, and the brilliant reflexion at the surfaces of some of the metals, are capable of exhibiting some of the appearances of colour as if the

* Art. CHROMATICS, Supp. Encyc. Brit. p. 157.

light concerned were divided into two portions, the one partially reflected in the first instance, the other beginning to be refracted, and caused to return by the continued operation of the same power. The original interval appears to be extremely minute, but is capable of being increased by a repetition of similar reflexions as well as obliquity of incidence."

In a letter which I received from this eminent philosopher, dated March 25th 1816, he thus modifies an objection which he had previously made to my opinion, that the phenomena were owing to the interference of the light which had entered the surface with that which had suffered partial reflexion.

"The light which you suppose to have entered a little way into a reflecting surface, in the case of total reflexion, is singularly circumstanced with regard to the objection I mentioned in my last letter. I did not like the idea of supposing a surface of any kind to contain a finite space: but, in fact, if your theory should be confirmed, this objection might be greatly diminished by the consideration, that the thickness of the surface would still be like an infinitesimal of a different order from the interval corresponding to its apparent effect, being the versed sine of a curve of which that small interval is the arc, and possibly in a circle of curvature not very minute."

In continuing my experiments on this subject, I found that the colours of total reflexion did not rise in the scale by successive reflexions; and as they modified the tints of crystallized bodies by adding to, or subtracting from, them a given portion of a tint, I announced in the end of 1816, in the Journal of the Royal Institution, that I had discovered "a new species of moveable polarization, in which the complementary tints never rise above the white (the blueish white) of the first order, by the successive application of the polarizing influence*." I determined, experimentally, the angles at which this tint was successively produced and destroyed, and thus discovered some of the leading properties of total reflexion, before, I believe, M. FRESNEL had made any experiments on the subject. It was he, however, who ascertained that this new species of polarization was circular polarization; and it is impossible to speak too highly of the ingenuity and talent which he exhibited in that difficult inquiry.

This view of the phenomena of total reflexion unsettled the opinions which I

* Journ. Roy. Inst. vol. iii. p. 213.

had entertained respecting the action of metals, and I was thus led to revise and extend the unpublished experiments which I had made on the subject.

In order to ascertain the effect of a single metallic surface, I took a crystallized plate of glass whose central tint was the blueish white of the first order, and positive like sulphate of lime. This tint varied from a quarter of a tint in value down to zero. The primitive ray was polarized +45°, and the plate of steel was horizontal. This ray was received at an incidence near 90°, and the principal section of the analysing prism was in the plane +45°, while the length of the plate of glass was fixed perpendicular to the plane +45°, or to the principal section of the prism, so as to move along with it.

At an incidence of 88° the metallic action destroyed the action of the equivalent crystallized plate when the section of the analysing prism was turned from +45° to +38°.

At an incidence of 83½° the same effect was produced when the same section was turned into the plane +22½°.

And at an angle of 75°, viz. the maximum polarizing angle, the compensation took place when the axis of the crystal had moved round 45°.

In like manner, at an angle of 60° the compensation took place when the axis of the crystal was turned round 45° + 22½°, or − 22½°; and,

At an angle of incidence of 40° the compensation was effected when the axis of the crystal had turned round 45° + 37°, or into the plane − 37°. The same results are obtained when the light falls on the metal before it passes through the crystal.

Hence it follows, that at the maximum polarizing angle the effect of the equivalent crystal placed in azimuth 45° to the plane of primitive polarization, is compensated by the action of the metallic surface, while at greater angles of incidence the compensation is effected in azimuths less than 45°; and, at less angles of incidence, in azimuths greater than 45°.

When the reflexion from the metal is made in a plane perpendicular to the meridian, the opposite effect is produced.

The angles at which the compensation takes place in the preceding experiments are obviously such, that calling R the angle of rotation of the axis of the crystal, it has always to i the angle of incidence the same relation as in the formula, $\tan(45° - R) = \frac{\cos(i + i')}{\cos(i - i')}$.

Hence we are led to the important conclusion, that the pencil which enters the metal follows the changes of polarization of the partially reflected pencil, which is regulated by the same law as in transparent bodies.

It now became interesting to examine the effect produced by the joint action of the metal, and an equivalent crystal, in changing the plane of polarization of the restored ray. The following are the results with different metals at the maximum polarizing angle.

Metals.	Position of the Plane of Polarization.	Rotation effected.
Silver (pure)	$+42^\circ$	3°
Copper	$+36\frac{1}{2}$	$8\frac{1}{2}$
Mercury	$+35$	10
Platina	$+34$	11
Speculum metal	$+32$	13
Steel	$+30\frac{1}{2}$	$14\frac{1}{2}$
Lead	$+26$	19
Galæna	$+17\frac{1}{2}$	$27\frac{1}{2}$

These metals follow the same order in their action upon the plane of polarization that they hold in the Table in page 294, though in reference to the rotation actually produced in both cases the order is inverted.

The preceding Table points out in a very instructive manner the difference between the action of a metallic surface and an equivalent crystallized film. When two metallic surfaces act together, the plane of polarization of the restored ray is invariably thrown beyond the plane of reflexion; whereas in the combination of a crystallized film with a metallic surface, the same plane never reaches the plane of reflexion, the plane having always a negative position in the former case, and a positive one in the latter. Thus in two reflexions from silver at 73°, the primitive ray polarized $+45^\circ$ has its plane of polarization changed into $-39^\circ 48'$, whereas in the combination of one reflexion from silver with the crystallized film, the plane is changed only into $+42^\circ$.

In order to determine the law of the metallic action at different incidences and with different numbers of reflexions, I interposed between the eye and the metal, which was silver, a plate of calcareous spar, which exhibited its uniaxal system of rings.

Fig. 2.

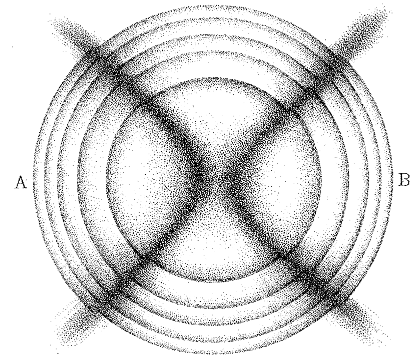


Fig. 1.

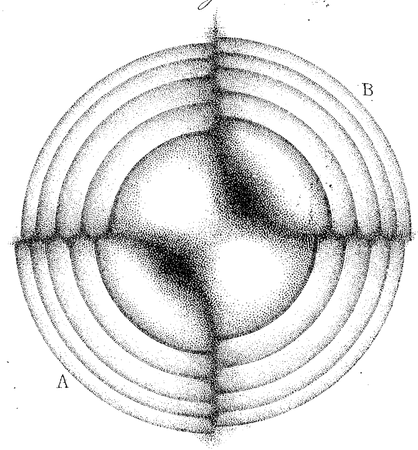
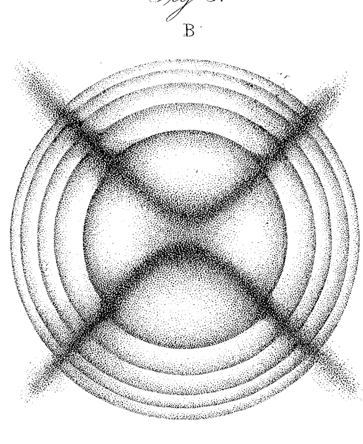


Fig. 3.



The influence of the metal in modifying the rings was a maximum at 73°, as shown in Plate XI. fig. 1. exactly as if they had been crossed by a positive crystalline film which polarized a quarter of a tint, or the pale blueish white of the first order, and whose axis was situated in a plane +45°, or that which bisects the planes of the two pencils oppositely polarized by the metal. The influence of the metal, or the tint which it polarizes, diminishes gradually from 73° to 90°, where it vanishes, and consequently where the rings recover their symmetry and their tints. At this limit the position of the axis of the equivalent film is A B (fig. 2.), a line still bisecting the planes of the two oppositely polarized pencils. In fig. 2. the rings are not represented of their own shape, but just as they are beginning to be invaded by the metallic action as at an incidence of 86° or 87°. At incidences from 73° to 0° the opposite effect takes place, the rings recovering their symmetry at 0°, and the position of the axis of the equivalent film being now vertical, and bisecting the planes of the two oppositely polarized pencils. The form of the rings before they recover their symmetry is shown in fig. 3.

At all intermediate angles of incidence the axis A B has intermediate positions; and calling A the inclination of the axis to the plane of reflexion, we shall have $A = \varphi + 45°$,

φ being positive or + from 90° to 73°, and negative or − from 73° to 0°.

The intensity of the metallic tint, so to speak, or of the positive equivalent plate T, will be $$T = \tfrac{1}{4}\,\frac{P}{90} = \frac{2\,R}{360} = \left(\frac{45° - \varphi}{180}\right).$$

Hence we see the error of the proposition hitherto maintained, that an increase of incidence, reckoning from the perpendicular, produces the same effect as an increase of thickness in thin crystallized plates.

When the rings are combined with two reflexions at 73° in silver, or 75° in steel, they do not suffer the slightest change, the principal section of the prism being placed in the plane −39° 48′ with silver, and −17° with steel. By two reflexions, however, between 73° or 75° and 90°, an effect is produced on the rings which increases gradually in silver from 73° to 82° 30′, and diminishes from 82° 30′ to 90°. At 82° 30′ the effect is the same as after a single reflexion at 73°; for since four reflexions at 82° 30′ restore the elliptically polarized ray, two reflexions at the same angle must have produced complete elliptical polarization. At angles between 82° 30′ and 90° the pencil is only partially polarized elliptically; whereas from 82° 30′ to 73° the light has been more than elliptically polarized, the restoration of it having been begun during the second reflexion. Hence, in order to determine the phase for any angle between 82° 30′ and 90°, we must take the sum of the phases for each reflexion, or 2 P; whereas between 82° 30′ and 73° we must take the excess of the sum of the two phases above 90° or 90° − 2 P. In both cases the pencil has suffered a partial elliptic polarization;—in the former, from the sum of the actions of the two reflexions, and in the latter, from their unbalanced actions. The very same effects take place between 73° and 57° 16′, the other maximum, as between 73° and $82\frac{1}{2}$°; and between 57° 16′ and 90°, as between $82\frac{1}{2}$° and 90°.

In the case of three reflexions there are two points or nodes of restoration, viz. 78° 8′ and 66° 35′, the maxima being at 85° 6′, 73°, and 48° 38′, at each of which points the phase is 90°. At 73° the second reflexion restores the ray elliptically polarized by the first reflexion, and the third reflexion again produces elliptic polarization. At 85° 6′ and 48° 38′, six reflexions produce a restoration of the pencil, and consequently three reflexions must have polarized the pencil elliptically with a phase of 90°. From 85° 6′ to 90°, and from 48° 38′ to 0°, the pencil has been only partially elliptically polarized, and the phase at any angle between these will be 3 P. At any angle between 48° 38′ and 85° 6′, the phase will be $\overline{2 \times 90} - 3\,\mathrm{P}$.

In general, calling n the number of reflexions, the phase between 90° and the nearest maximum, and between 0° and the nearest maximum, will be $n\,\mathrm{P}$, while at all other angles of incidence it will be $(\overline{n - 1} \times 90) - n\,\mathrm{P}$.

In order to give a general view of the number of points of restoration, and of the other phenomena which take place after different numbers of reflexions, I have drawn up the following Tables.

Table I. Showing the numbers of reflexions from silver at which elliptically polarized light is restored to a single plane of polarization, with the corresponding angles of incidence, and the position of the plane of restoration in relation to the plane of reflexion, computed for 20 reflexions.

(For angles less than the maximum polarizing angle.)

No. of Reflexions.		Integer Multiples.	Angle of Restoration. °	′	No. of Reflexions.		Integer Multiples.	Angle of Restoration. °	′
−2	2	+4−6+8−10+12−14+16−18+20	71	0	$3\frac{4}{5}$	3.8	+19	58	15
$2\frac{1}{9}$	2.111	+19	71	42	− 4	4.0	+8−12+16−20	57	16
$2\frac{1}{8}$	2.125	+17	71	32	$4\frac{1}{4}$	4.25	+17	55	54
$2\frac{1}{7}$	2.143	+15	71	23	$4\frac{1}{3}$	4.333	+13	55	29
$2\frac{1}{6}$	2.167	−13	71	8	$4\frac{1}{2}$	4.5	−9+18	54	42
$2\frac{1}{5}$	2.200	+11	70	48	$4\frac{2}{3}$	4.667	−14	53	54
$2\frac{2}{9}$	2.222	−20	70	34	$4\frac{3}{4}$	4.75	−19	53	31
$2\frac{2}{8}$	2.25	−9+18	70	17	+ 5	5	+10+15+20	52	27
$2\frac{2}{7}$	2.286	−16	69	53	$5\frac{1}{3}$	5.333	−16	51	5
$2\frac{2}{6}$	2.333	+7+14	69	29	$5\frac{1}{2}$	5.5	−11	50	27
$2\frac{3}{8}$	2.375	−19	69	3	$5\frac{2}{3}$	5.667	−17	49	49
$2\frac{2}{5}$	2.4	−12	68	59	− 6	6	+12−18	48	38
$2\frac{3}{7}$	2.428	−17	68	33	$6\frac{1}{3}$	6.333	+19	47	23
$2\frac{1}{2}$	2.5	−5+10−15+20	67	54	$6\frac{1}{2}$	6.5	−13	46	57
$2\frac{4}{7}$	2.571	−18	67	14	$6\frac{2}{3}$	6.667	−20	46	32
$2\frac{3}{5}$	2.6	+13	66	58	+ 7	7	+14	45	35
$2\frac{2}{3}$	2.667	−8+16	66	25	$7\frac{1}{2}$	7.5	−15	44	13
$2\frac{5}{7}$	2.714	+19	66	0	− 8	8	+16	43	0
$2\frac{3}{4}$	2.75	−11	65	45	$8\frac{1}{2}$	8.5	+17	41	52
$2\frac{4}{5}$	2.8	−14	65	23	+ 9	9	+18	40	51
$2\frac{5}{6}$	2.833	+17	65	9	$9\frac{1}{2}$	9.5	−19	39	51
$2\frac{6}{7}$	2.857	−20	65	0	−10	10	+20	39	0
+3	3	+6+9+12+15+18	63	43	+11	11		37	15
$3\frac{1}{6}$	3.167	−19	62	29	−12	12		35	50
$3\frac{1}{5}$	3.2	−16	62	15	+13	13		34	33
$3\frac{1}{4}$	3.25	−13	61	55	−14	14		32	30
$3\frac{1}{3}$	3.33	−10	61	20	+15	15		32	15
$3\frac{2}{5}$	3.4	−17	60	53	−16	16		31	17
$3\frac{1}{2}$	3.5	−7+14	60	15	+17	17		30	30
$3\frac{3}{5}$	3.6	−18	59	38	−18	18		29	42
$3\frac{2}{3}$	3.667	+11	59	13	+19	19		28	56
$3\frac{3}{4}$	3.75	−15	58	42	−20	20		28	10

Table II. Showing the numbers of reflexions from silver at which elliptically polarized light is restored to a single plane of polarization, with the corresponding angles of restoration, and the position of the plane of restoration in relation to the plane of reflexion, computed for 20 reflexions.

(For angles greater than the maximum polarizing angle.)

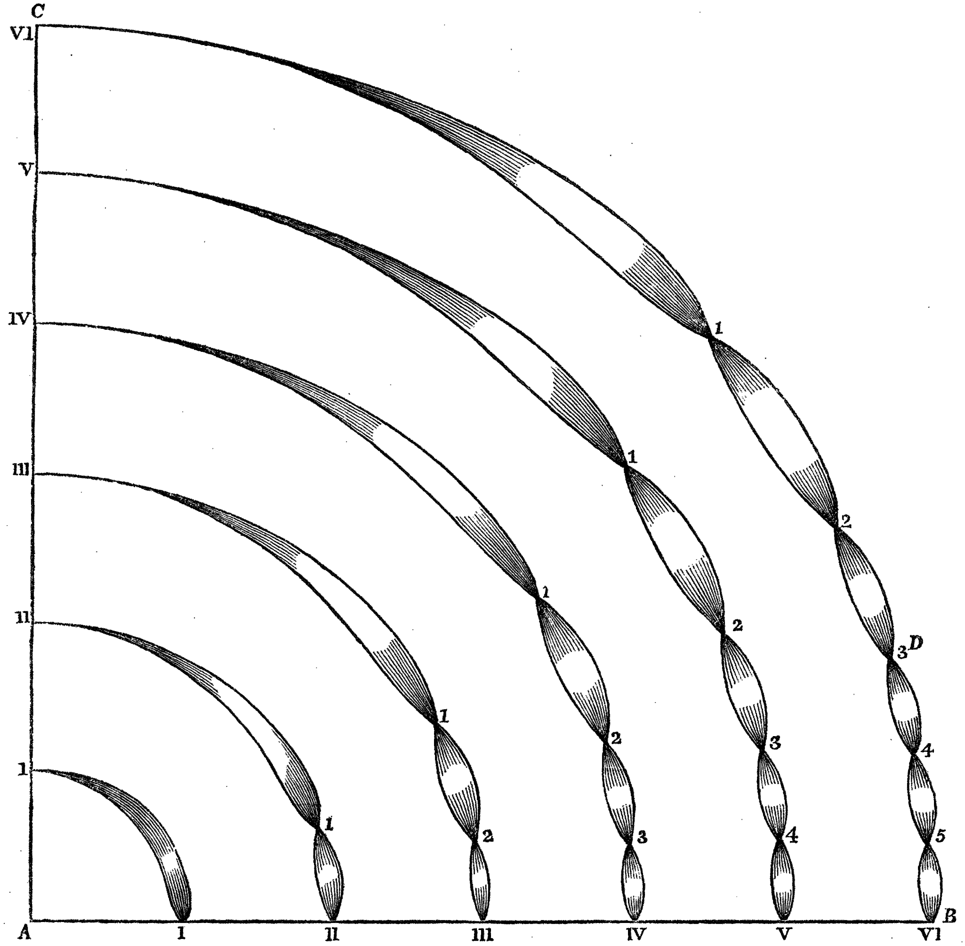
No. of Reflexions.		Integer Multiples.	Angle of Restoration.	No. of Reflexions.		Integer Multiples.	Angle of Restoration.
			° ′				° ′
−2	2	+4−6+8−10+12−14+16−18−20	73 0	$3\frac{4}{5}$	3.8	−19	82 8
$2\frac{1}{9}$	2.111	−19	74 9	− 4	4	+8−12+16−20	82 30
$2\frac{1}{8}$	2.125	+17	74 18	$4\frac{1}{4}$	4.25	+17	82 58
$2\frac{1}{7}$	2.143	−15	74 28	$4\frac{1}{3}$	4.333	−13	83 16
$2\frac{1}{6}$	2.167	+13	74 44	$4\frac{1}{2}$	4.5	+9+18	83 23
$2\frac{1}{5}$	2.200	−11	75 0	$4\frac{2}{3}$	4.667	−14	83 38
$2\frac{2}{8}$	2.222	−20	75 12	$4\frac{3}{4}$	4.75	+19	83 45
$2\frac{1}{8}$	2.256	+9+18	75 26	− 5	5	+10−15+20	84 5
$2\frac{2}{7}$	2.286	−16	75 43	$5\frac{1}{3}$	5.333	−16	84 27
$2\frac{2}{6}$	2.333	−7+14	76 2	$5\frac{1}{2}$	5.5	+11	84 38
$2\frac{3}{8}$	2.375	+19	76 19	$5\frac{2}{3}$	5.667	−17	84 48
$2\frac{2}{5}$	2.4	−12	76 33	− 6	6	+12−18	85 6
$2\frac{3}{7}$	2.428	+17	76 44	$6\frac{1}{3}$	6.333	−19	85 22
$2\frac{1}{2}$	2.5	+5+10+15+20	77 13	$6\frac{1}{2}$	6.5	+13	85 30
$2\frac{4}{7}$	2.571	−18	77 38	$6\frac{2}{3}$	6.667	−20	85 36
$2\frac{3}{5}$	2.6	−13	77 48	− 7	7	+14	85 49
$2\frac{2}{3}$	2.667	−8+16	78 38	$7\frac{1}{2}$	7.5	+15	86 7
$2\frac{5}{7}$	2.714	−19	78 23	− 8	8	+16	86 21
$2\frac{3}{4}$	2.75	+11	78 33	$8\frac{1}{2}$	8.5	+17	86 35
$2\frac{4}{5}$	2.8	−14	78 47	− 9	9	+18	86 46
$2\frac{5}{6}$	2.833	+17	78 57	$9\frac{1}{2}$	9.5	+19	86 56
$2\frac{6}{7}$	2.857	−20	79 4	−10	10	+20	87 5
−3	3.0	+6−9+12−15+18	79 40	−11	11		87 20
$3\frac{1}{6}$	3.167	+19	80 37	−12	12		87 35
$3\frac{1}{5}$	3.2	−16	80 24	−13	13		87 46
$3\frac{1}{4}$	3.25	+13	80 34	−14	14		87 56
$3\frac{1}{3}$	3.333	−10+20	80 50	−15	15		88 4
$3\frac{2}{5}$	3.4	−17	81 2	−16	16		88 11
$3\frac{1}{2}$	3.5	+7+14	81 19	−17	17		88 18
$3\frac{3}{5}$	3.6	−18	81 35	−18	18		88 24
$3\frac{2}{3}$	3.667	−11	81 45	−19	19		88 28
$3\frac{3}{4}$	3.75	+15	81 57	−20	20		88 33

The first column of the preceding Tables shows the smallest number of reflexions at which a pencil of elliptically polarized light is restored to a single plane of polarization at the angle contained in the fourth column; and consequently the half of these numbers is the number of reflexions at which light is elliptically polarized at the same angle. Thus at three reflexions the ray is restored to a single plane of polarization at 63° 43′, and 79° 40′, and consequently at $1\frac{1}{2}$ reflexion it is elliptically polarized at that angle. This is easily understood when the number of reflexions is an integer; but it requires some explanation when the number is partly fractional. It has been already stated, in page 301, that elliptical polarization may be completed at any fractional part of a reflexion; and since it begins to be restored the instant the polarization is complete, and again begins to be elliptically polarized after every restoration, the points of restoration may take place in the middle of a reflexion; and though we cannot possibly examine what takes place at these points, yet the effect must appear when the fractional number of reflexions in the first column has been repeated so many times as to become a whole number. Thus a ray elliptically polarized by $1\frac{1}{3}$ reflexion will be restored to a single plane at $2\frac{2}{3}$ reflexions at the same angle. It will also be restored at $2\frac{2}{3} \times 2 = 3\frac{1}{3}$, and at $2\frac{2}{3} \times 3 = 8$, in which case its restoration will be seen at the eighth reflexion at the same angle; and also at the sixteenth and twenty-fourth, &c. In this case the phase P will be $\frac{90°}{1\frac{1}{3}} = 67\frac{1}{2}°$, $R = 33° \ 45'$, and $\varphi = 11° \ 15'$, from which we deduce the angles of incidence to be 63° 43′, and 79° 40′. In order to ascertain the existence of these points of restoration, I made the experiment at five and seven reflexions as multiples of $2\frac{1}{2}$ and $2\frac{1}{3}$, and I found the angles to be for five reflexions 68°, and for seven reflexions 70°, in place of 67° 54′, and 69° 29′, as computed from the formula.

The numbers in the third column, with the signs + and −, are the integer multiples of those in the first column, and show the number of reflexions at which the elliptically polarized light is restored, the numbers being carried the length of twenty reflexions. The sign + shows that the plane of the restored ray is to the right, and the sign − that it is to the left of the plane of reflexion. In order to determine the sign of the restored ray, we must consider that in the same quadrant the signs necessarily alternate. Now at 73°, the maximum polarizing angle, the signs are −2, +4, −6, +8, −10, +12, &c.; and I have also found that all the integer numbers in column 1st, Table I. have their signs + or positive, as +3, +5, +7, +9, &c., and all the even numbers their signs − or negative, as −4, −6, −8, −10, &c.; whereas in Table II. all the integer numbers are negative whether odd or even, thus, −3, −4, −5, −6, &c.

By setting out therefore from these points, and attending to the alternation of the signs, it is easy to determine for any number of reflexions its proper signs, whether it is a multiple of an integer or of a mixed number.

In order to illustrate this Table, I have here projected some of its results as far as six reflexions. The concentric arches I I, II II, &c. represent the quadrant



of incidence for one, two, &c. reflexions, B being the point of 90°, and C that of 0° of incidence. The point D or the line A D is the point or line of maximum polarization, viz. 73° for silver; and the figures 1, 2, 3, 4, 5, &c. show the points or nodes, and their distances from C, the angles of restoration. The loops or double curves lying between the points 1, 2, 3, are drawn to give an idea of the intensity of the elliptic polarization, which has its minimum at 1, 2, 3, &c. and its maximum at intermediate points. These points of maximum intensity do not bisect the loops, or are not equidistant from the minima 1, 2, &c.; but such is their relation to them, that the maximum for n reflexions is the minimum for $2n$ reflexions corresponding to the same angle. Thus the maximum for one reflexion, viz. 73°, is the minimum for two reflexions; and the maxima for two reflexions, viz. 82° 30′ and 63° 43′, are the minima for four reflexions. The maximum may be found directly by computing the angle of incidence, which corresponds to a phase intermediate between the two minima, within which the maximum lies.

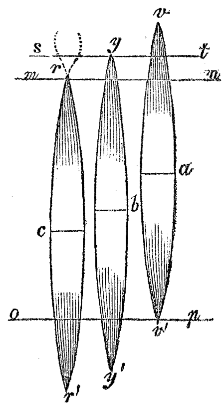
Having thus determined the various points of the quadrant, at which elliptic polarization is produced, and at which it is destroyed, after any number of reflexions; and also the position of the plane of the restored ray, I shall proceed to investigate the cause of those brilliant complementary colours which accompany these phenomena.

As all transparent bodies have different values of their maximum polarizing angle, appropriate to the index of refraction for each colour of the spectrum, it is reasonable to suppose that as elliptic polarization is effected at the maximum polarizing angle, this angle would vary for the differently coloured rays. That this is the case may be easily proved by observing the angles of restoration for homogeneous light after two reflexions. In silver the difference of the angles for red and blue light is about 5° in the sun's rays; so that calling 73° the maximum polarizing angle for the mean yellow ray, the angle will be $70\frac{1}{2}$° for blue, and $75\frac{1}{2}$° for red light. Hence if we examine a pencil of white light twice reflected at $70\frac{1}{2}$°, and place the principal section of the analysing prism in the plane −39° 48′, the blue rays will disappear and the red will remain visible. In like manner, at an angle of 75° 30′ the red will disappear, and the complementary blue will be visible; while at an angle of 73° the yellow will disappear and red and blue will be seen together, one on each side of the place where the yellow has vanished. At angles of incidence greater than $75\frac{1}{2}$° and less than $70\frac{1}{2}$°, and also at intermediate angles, the blue or the red wil still predominate in the pencil, the blue being in excess at all angles greater than 73°, and the red in excess at all angles less than 73°. Such are precisely the phenomena which take place, as will appear from the following Table.

Angle of Incidence of the two Reflexions.	Colours with ordinary Light.
63°	Very pale yellow, growing whiter at less incidences
64	Pale yellow.
65	Pale saffron yellow.
66	Saffron yellow.
67	Paler orange yellow.
68	Orange yellow.
69	Reddish orange.
70	Tile red.
$70\frac{1}{2}$	Vermilion red.
71	Scarlet.
72	Bright pink.
73	Dark pink.
74	Deep China blue.
75	Indigo.
$75\frac{1}{2}$	Pure bright blue.
76	Paler blue.
77	Whitish blue.
78	Blueish white, growing white at greater angles.

It is obvious from what has been already stated, that with homogeneous yellow light the pencil will not vanish in passing from 73°, where it is evanescent, to 90°, and 0° where it is also evanescent; but the intensity of the extraordinary pencil of the analysing rhomb will increase from 0° to half the reflected light, from 73° to $82\frac{1}{2}$°, and from 73° to 57° 16′, and will decrease from the same points to 90° and 0°. The same is true of the red and blue rays, the former having its maximum intensity at an angle greater than $82\frac{1}{2}$° and greater than 57° 16′, and the latter at an angle less than $82\frac{1}{2}$° and less than 57° 16′.

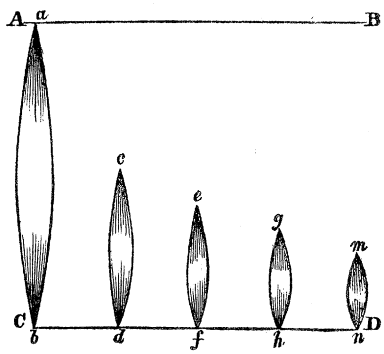
In order to ascertain the phenomena in homogeneous light, let us suppose that polarized yellow light suffers four reflexions from silver, and let us consider what should take place in the loop 2, 3 of the quadrant IV, IV. (See Fig. p.318.) At the node 2, or 73°, the inclination of the restored pencil is +31° 52′, and at the node 3, or 82° 30′, it is −37° 22′, and the point of maximum between 2 and 3 is at 78° 8′. If at 73° we place the principal section of the analysing prism in the plane $+31° 52'$ the extraordinary ray will vanish, and the light will pass into the ordinary image; and if at 82° 30′ we place it in $-37° 22'$, the same effect will be produced. At 74° a small portion of light will pass into the extraordinary image, and this portion will gradually increase to 78° 8′, the principal section of the prism having been turned round gradually from $+31° 52'$ to 0°, as described in page 291. The ordinary and extraordinary images now approach most to equality, and they vary in intensity according to the same law in passing from 78° 8′ to 82° 30′, the axis of the prism having now come into the plane $-37° 22'$. The very same phenomena take place with red and blue light, only the points of restoration and the maximum occur at different angles of incidence, so that the spaces between the minima have different lengths for the differently coloured rays. These spaces or loops, therefore, will overlap each other, as will be understood from the annexed diagram, where they are shown separately, $r\,r'$ being the red loop, $y\,y'$ the yellow, $v\,v'$ the violet one, the points r, y, v, r', y', v' the minima or nodes, and a, b, c the maxima. When these loops are viewed superposed as when they form white light, then the tint in the extraordinary image will be white, minus the three quantities of light that have disappeared from the extraordinary ray. At the line $m\,n$, passing through the node of the red loop, the red will have vanished, and the mixture of the yellow and the violet which remains will constitute a greenish blue pencil, decreasing in its blue tint towards a, and becoming pink, and then red towards $t\,s$, in consequence of part of the light of the other red loop above r now passing into the extraordinary ray. At v and at v', where the violet disappears, the mixture of the yellow and the red will form an orange pencil, which will be reddest at v and v', and shading off to white at a. At the line $s\,t$ the yellow vanishes, and across the upper part of the luminous disc, there will be light with an excess of red, and across the lower part of it, light with an excess of blue. This takes place with even numbers of reflexions; with odd numbers the blue light is uppermost and the red undermost.



The phenomena of colour, as seen by white light, vary greatly with the number of reflexions, both with respect to the depth of the colours themselves and the rapidity of their changes. In order to investigate the nature of these variations, let us consider what will take place at 2, 4, 6, 8, and 10 reflexions from silver in the loops above and adjacent to 73° the maximum polarizing angle. The following are the numbers which regulate the phenomena.

Fig. 6.	No. of the Reflexions.	Nature of the Reflexions.	Limits of the Loops.	Length of the Loops.	Inclination of the Plane, or φ.
			° ° ′	° ′	° ′
a b	2	First of the series	73 — 90 0	17 0	39 48
c d	4	First of the series	73 — 82 30	9 30	37 22
e f	6	Multiple of 3	73 — 79 40	6 40	32 25
g h	8	Multiple of $2\frac{2}{3}$	73 — 78 8	5 8	27 53
m n	10	Multiple of $2\frac{1}{2}$	73 — 77 13	4 13	24 16

This Table may be illustrated by the annexed diagram, where A B passes through the incidence of 90°, and C D through that of 73°, the points *m, g, e, c, a* corresponding respectively with the incidences of 77° 13′, 78° 8′, 79° 40′, and 82° 30′, or those at which the ray is restored by 10, 8, 6, and 4 reflexions. The curvilineal spaces *a b, c d, e f, g h,* and *m n,* are the loops already referred to, whose breadths represent the intensity of the extraordinary ray, which is a minimum at the nodes *a, c, e, g, m,* and *b, d, f, h, n,* and reaches its maximum near the middle of the loops.



If the image reflected from the silver is a circular disc of white light of a given magnitude, then by two reflexions at 73°, or at the point *b* the extraordinary image will be red above and blue below, when the principal section of the analysing prism is in the plane — 39° 48′; but these colours will be very faint, as the disc occupies but a small part of the loop *a b*. The disc indeed may be made so small, that the extraordinary image will entirely disappear in this loop. In this case the ordinary image will be white, as all the reflected light will pass into it. At four reflexions the loop *c d* is little more than one half of *a b*, and consequently the light will vary much more rapidly from *d* to the maximum. When the analysing prism has its principal section in the plane —37° 22′, the extraordinary image at *c* will be coloured with red light above and blue below; and when it is in the plane + 31° 52′, the extraordinary image at *d* will be similarly coloured: The colours will be much brighter than in the case of two reflexions, and consequently the extraordinary image will not vanish. The consequence of this is, that the ordinary image is not white as before, but yellow, because a considerable portion of red and blue light are left in the extraordinary image.

As the number of reflexions increase, and the loops $e f$, $g h$, &c. diminish, the disc will occupy a greater proportion of the whole loop, and the red and blue colours with which it is crossed grow brighter and brighter, and come closer and closer to their line of junction in the middle of the disc. Hence a greater quantity of red and blue light is left out of the ordinary image, which on this account becomes yellower and yellower, and at last of a greenish hue.

In order to determine the position of the principal section of the analysing prism, when the extraordinary image is a minimum for any angle of incidence α, and any number of reflexions, let $\psi, \chi =$ the inclinations of the plane of polarization of the restored ray at the nodes a, b; $m, n =$ the inclinations or values of φ in the formula $\tan \varphi = \frac{\cos (i + i')}{\cos (i - i')}$ suited to the angles of incidence at the nodes; $x =$ the inclination φ suited to the incidence α.

Now it is obvious that at the one node, the position of the principal section of the analysing prism, when the extraordinary image is a minimum, is $+ \psi$, and that it gradually changes to 0° and then passes to $- \chi$, thus undergoing a change equal to $\psi + \chi$, while the inclination φ varies by a quantity equal to $m - n$. Hence calling I the inclination of the principal section to the plane $+ \psi$ at the angle of incidence α, we have $m - n : \psi + \chi = m - x : \mathrm{I}$.

$$\text{Hence } \mathrm{I} = \psi + \chi \left(\frac{m - x}{m - n}\right)$$

$$\text{When } x = n, \ \mathrm{I} = \psi + \chi.$$

$$\text{When } x = \frac{m - n}{2}, \quad \frac{m - x}{m - n} = \tfrac{1}{2} \quad \text{and } \mathrm{I} = \frac{\psi + \chi}{2}.$$

When the nodes of the loop are on different sides of the maximum polarizing angle, which happens only in the middle loop of 3, 5, 7, &c. reflexions, then m and n have opposite signs, and consequently their difference is $m + n$, and, as in this case $m = n$, the formula becomes $\mathrm{I} = \psi + \chi \left(\frac{m - x}{2 m}\right)$.

It is impossible to determine the relative intensities of the ordinary and extraordinary image at any angle α, because this must depend on the relative intensities of the pencils by whose interference the elliptical polarization is produced. In silver these pencils approach to equality, but in steel and other metals they are very unequal.

Having thus shown how to determine the phenomena of elliptic polarization for any angle of incidence, for any number of reflexions, and for homogeneous light of any colour, I shall conclude this paper with some observations on a very remarkable anomaly which has presented itself in the course of this inquiry.

The phenomena which have been described, indicate very clearly that the angle of maximum elliptic polarization for one reflexion, or the angle of restoration after two equiangular reflexions, is the maximum polarizing angle of the metal, and consequently that its tangent is the index of refraction, as shown in the following Table*.

Names of Metals.	Angles of Maximum Polarization.	Index of Refraction.
Grain tin	78° 30′	4.915
Mercury	78 27	4.893
Galæna	78 10	4.773
Iron pyrites	77 30	4.511
Grey cobalt	76 56	4.309
Speculum metal	76 0	4.011
Antimony melted	75 25	3.844
Steel	75 0	3.732
Bismuth	74 50	3.689
Pure silver	73 0	3.271
Zinc	72 30	3.172
Tin plate hammered	70 50	2.879
Jewellers' gold	70 45	2.864

* This Table completely proves that the refractive index of metals cannot be deduced from their reflective power; for silver, which surpasses them all in reflective power, stands very low in refractive power. Mr. HERSCHEL has noticed the difference between the indices of refraction deduced by these two methods in the case of mercury, which he makes 5.829 as given by its reflective power, and 4.16 as given by its polarizing angle. He makes the index for steel 2.85. When we consider that metals reflect the light that enters their substance, it must be obvious that the quantity of light which

This conclusion is not opposed by any of the phenomena, when we consider merely the mean refrangible ray to which these numbers refer: but when we use homogeneous light, a very strange anomaly occurs. The maximum angle of elliptic polarization for red light in the case of silver is 75° 30′, and for blue light 70° 30′, giving

		Angle.
		° ′
Index of refraction for red light	3.866 . . .	75 30
——————— mean ray	3.271 . . .	73 0
——————— blue light	2.824 . . .	70 30

the order of the refrangibilities being inverted.

The perfect similarity between the action of metals, and the total reflexion of the second surfaces of transparent bodies, promised to throw light upon this difficulty. I accordingly examined the formula of Fresnel for total reflexion, where the phase P is thus expressed:

$$\text{Cos P} = \frac{2\,m^2 (\sin i)^4 - (m^2 + 1)(\sin i)^2 + 1}{m^2 + 1\,(\sin i)^2 - 1}.$$

From this formula it follows that when $m = 1.51$, and $i = 54°\ 37'$, P will be 45° for one reflexion, and consequently for two reflexions $2\ \text{P} = 90°$. If m increases as it does for blue light, then the phase will be 45° at an angle of incidence above 54° 37′, that is, the circular polarization of the pencil will take place at a greater angle of incidence for blue than for red light, which is the reverse of what takes place in metals. Upon making the experiment, however, with total reflexion, we shall find that the blue rays are circularly polarized by two reflexions at a less angle than the red rays, thus approximating the two classes of phenomena even with respect to this singular anomaly. Hence in order to accommodate M. Fresnel's formula to homogeneous light of different colours, let m be the index of refraction for the homogeneous ray, and d the difference between it and the mean index, then the formula for the phase P will become

$$\text{Cos P} = \frac{2\,(m \pm d)^2 (\sin i)^4 - \left((m \pm d)^2 + 1\right)(\sin i)^2 + 1}{\left((m \pm d)^2 + 1\right)(\sin i)^2 + 1}$$

they reflect is a function not only of their refractive power, but of their transparency, which will be proportional to the intensity of the reflected pencil that has entered the metal. If this is the case, the transparency will be proportional to the inclination of the plane of the restored ray after two reflexions at the maximum polarizing angle, and the order of the transparencies of the different metals will be that of the Table, p. 294. See Mr. Herschel's Treatise on Light, § 594, 845.

the sign + being used for the red or least refrangible rays, and — for the blue or most refrangible.

For the same reason, in calculating the phases of an elliptically polarized homogeneous ray by means of the formula $\tan \varphi = \frac{\cos (i + i')}{\cos (i - i')}$, we must determine i' from the formula $\sin i' = \frac{\sin i}{m \pm d}$, the sign + being used for the red or least refrangible, and — for the blue or most refrangible rays.

As the theoretical considerations upon which M. FRESNEL is said* to have constructed his formula, did not present to him the above anomaly, it would be in vain for me to seek an explanation of it. I may just mention, however, that at the second surfaces of bodies the angle of maximum polarization, or $\tan \frac{1}{m}$ is necessarily less for the least refrangible than for the mean rays, which is the reverse of what takes place at the first surface; and since the limit of total reflexion whose sine is $\frac{1}{m}$, or since the sphere of circular polarization commences sooner for the least than for the most refrangible rays, it might be expected that the angle of maximum circular polarization should be less for these rays, as I have found to be the case.

Although we do not understand the nature of the forces by which metals reflect the two oppositely polarized pencils, yet they act exactly like the second surfaces of transparent bodies when producing total reflexion. Setting out from a perpendicular incidence, the least refrangible rays begin to suffer the double reflexion sooner than the mean ray, and they sooner reach their maximum of elliptic polarization, thus exhibiting the inversion as it were of the spectrum, which we have noticed.

The theory of elliptic vibrations as given by FRESNEL, will no doubt embrace the phenomena of elliptic polarization; and when the nature of metallic action shall be more thoroughly examined, we may expect to be able to trace the phenomenon under consideration to its true cause.

ALLERLY, *February* 19*th*, 1830.

* I am acquainted with M. FRESNEL's formula only from the account given of it by Mr. HERSCHEL.